Towards the qualitative theory of large quantum coherent structures

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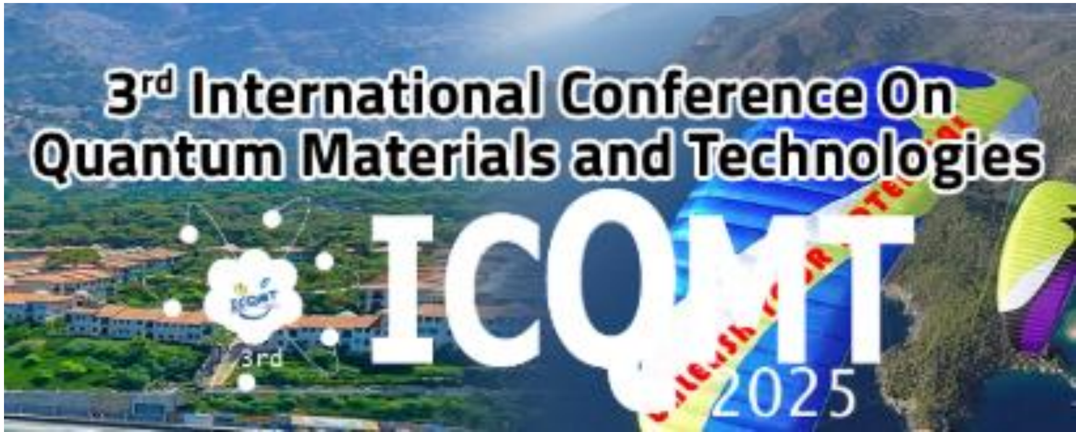

Abstract

- Design, characterization and optimization of large artificial quantum structures (e.g., practically useful quantum computers) is hindered by the fact that their efficient simulation by classical means is fundamentally impossible. On the other hand, important information about such systems can be obtained from a qualitative analysis of their "general case" behaviour. In particular, finding the universal dimensionless combinations of their parameters (figures-of-merit), which control transitions between qualitatively different regimes of operation, will help establish the desired parameters of the system with the use of scaled experiments and model calculations.

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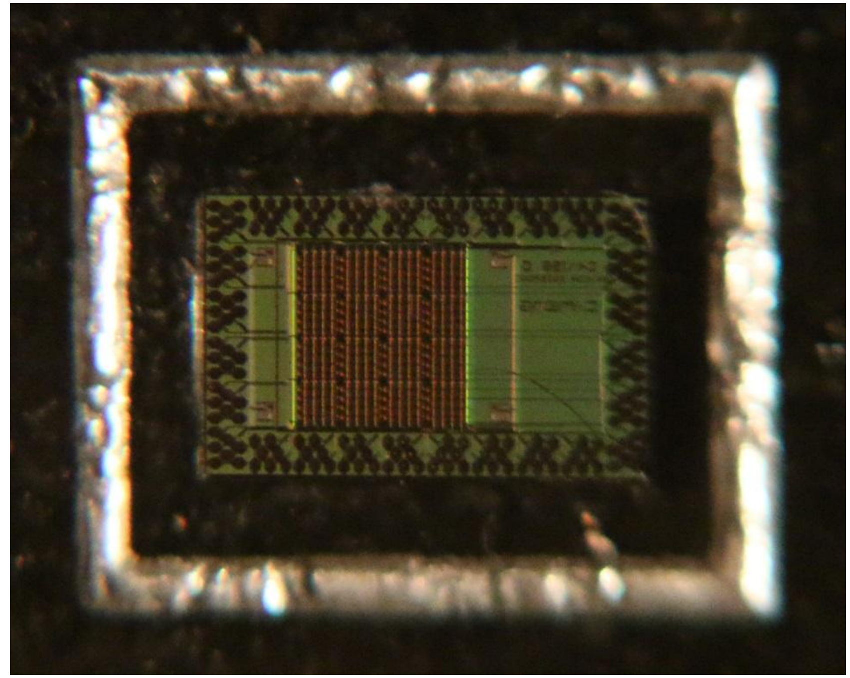
Prof. Shiro KAWABATA

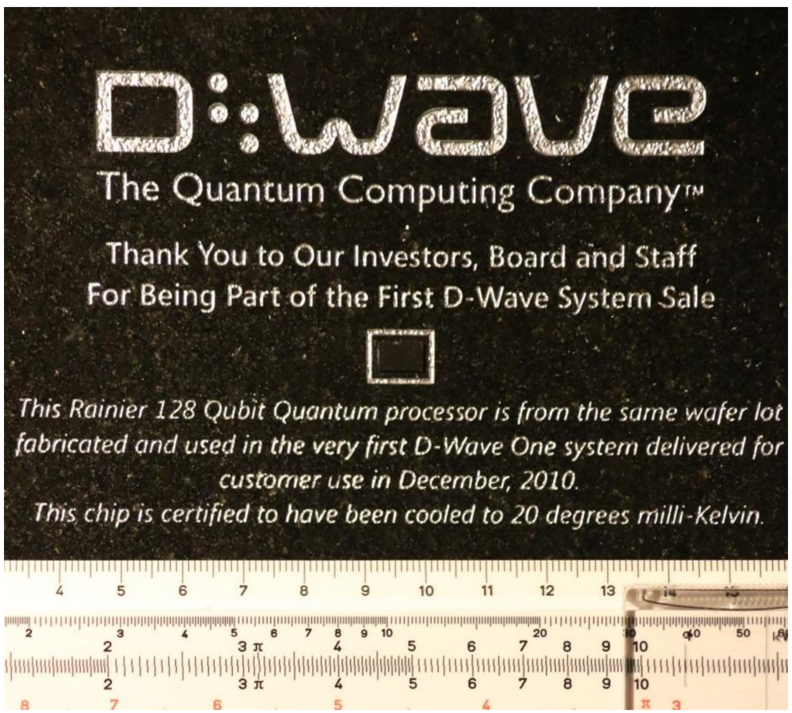
Dr. Alexander BALANOV

Dr. Mark GREENAWAY

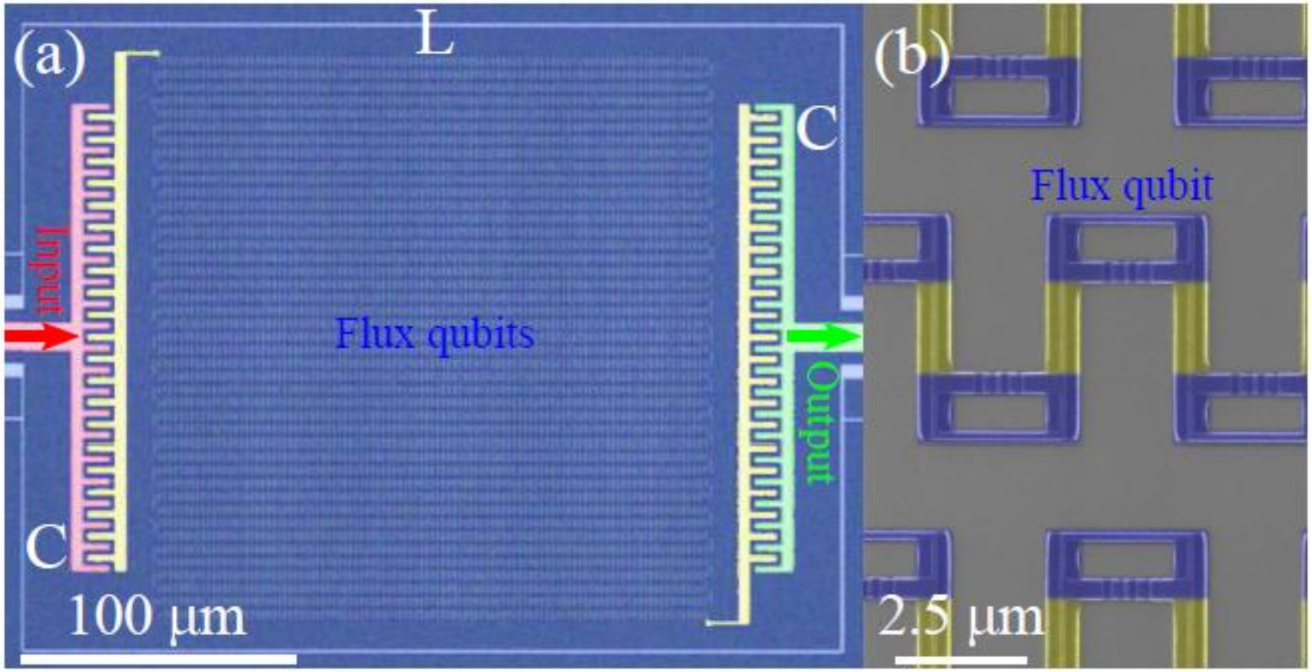
Current status

- Fabrication of multiqubit arrays with controlled macroscopic quantum coherence now possible
- Applications (part of “quantum technologies 2.0”):
  - Integrated quantum limited detection and image processing
  - Quantum optimization
  - Quantum simulation
  - Quantum communication

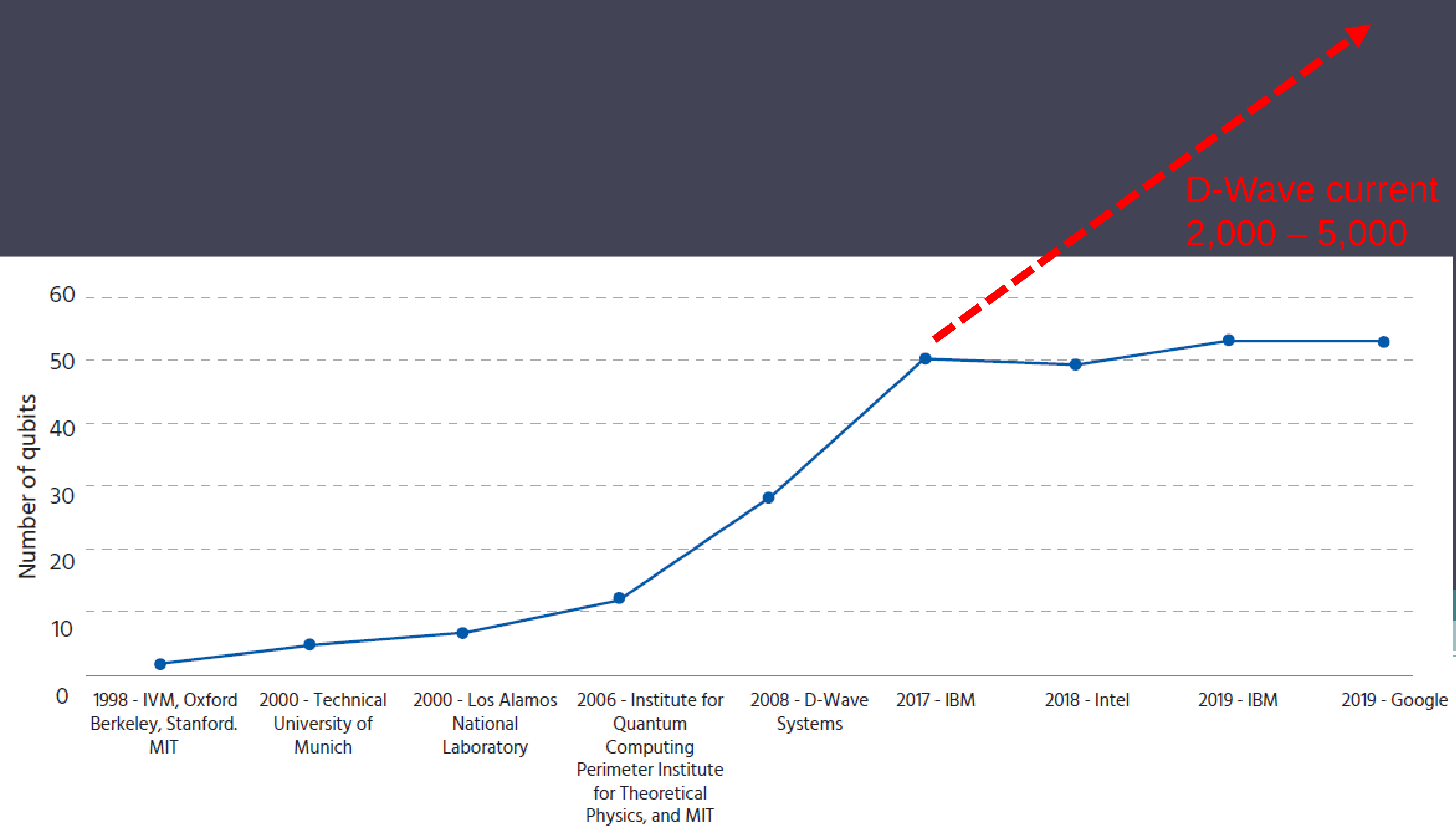



D::Wave
The Quantum Computing Company™
Thank You to Our Investors, Board and Staff
For Being Part of the First D-Wave System Sale
This Rainier 128 Qubit Quantum processor is from the same wafer lot fabricated and used in the very first D-Wave One system delivered for customer use in December, 2010.
This chip is certified to have been cooled to 20 degrees milli-Kelvin.

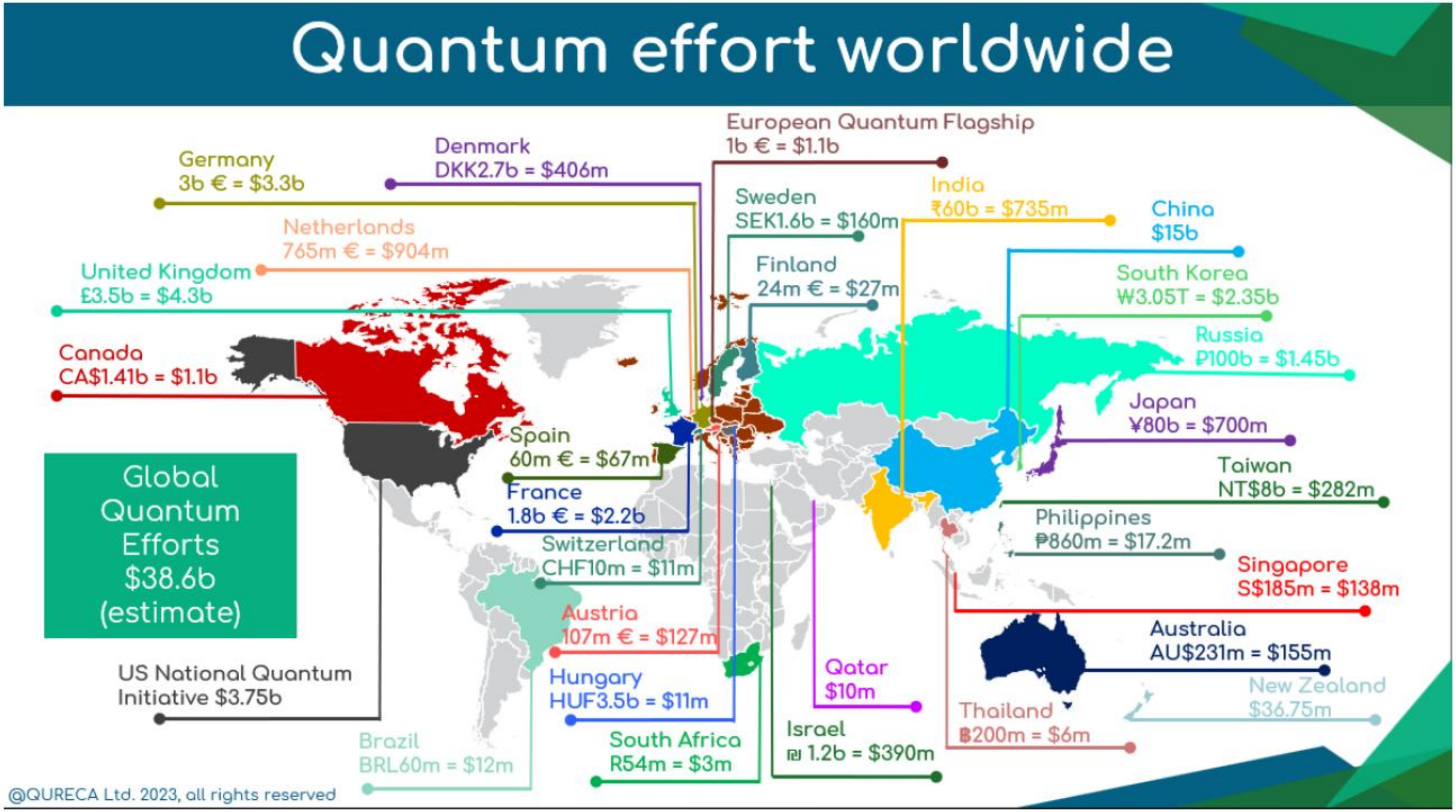
Collective coupling between an ensemble of 4300 flux qubits and a superconducting resonator



- Kakuyanagi et al., 2016

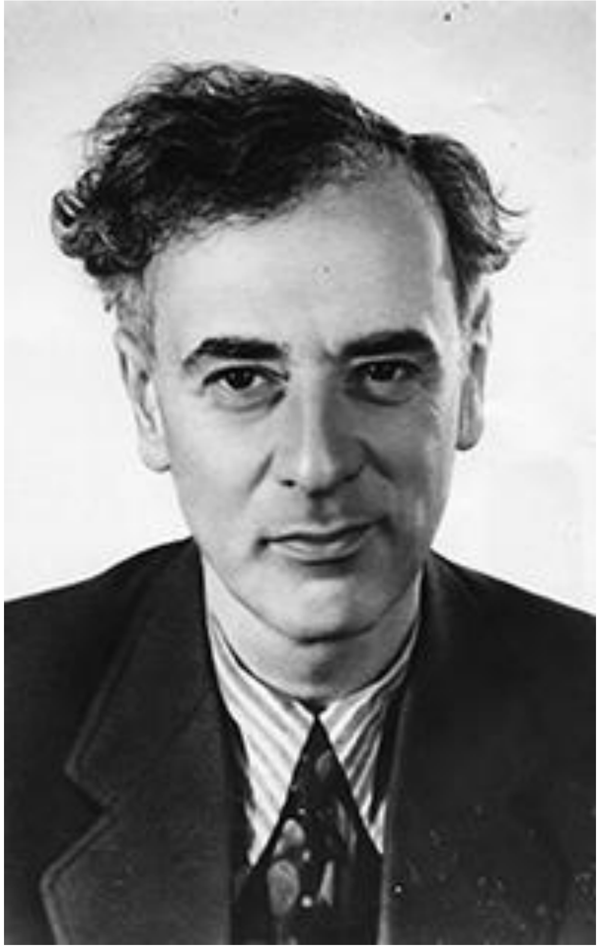


Sources: CB insights; IBM; Google; HINRICH FOUNDATION REPORT



- Scholten et al, arXiv:2401.16317v1

“Money is in the exponent.”



Lev Landau

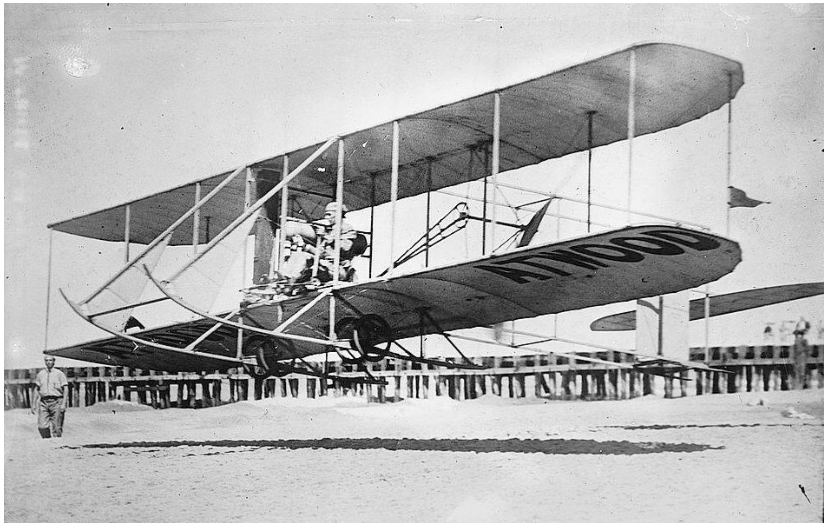
The fabrication and control of macroscopic artificial quantum structures, such as qubits, qubit arrays, quantum annealers and, recently, quantum metamaterials, have witnessed significant progress over the last 15 years and reached the point where **the existing theoretical and computational tools become inadequate** for predicting, analysing, and simulating their behaviour.

What now?

- Cannot build a Pentium™ with Steam Age technology
- Especially if you cannot tell whether it works or not

Drastic improvement in quality of superconducting qubits

- 1999 – <10 ns
- 2015 – >100 µs
- manipulation time - nanoseconds



…still short of what is needed for a universal digital quantum computer

Deceptive simplicity

- $\rho(\partial_t \boldsymbol{v} + (\boldsymbol{v} \cdot \nabla)\boldsymbol{v}) = -\nabla p + \mu \, \Delta \boldsymbol{v} + (\lambda + \mu)\nabla(\nabla \cdot \boldsymbol{v}) + \boldsymbol{f}$

- $i\hbar\partial_t \hat{\rho} = \left[\widehat{H}, \hat{\rho}\right] + \hat{\hat{L}}[\hat{\rho}]$


S

G

- It is absurd to search for a solution if it exists anyway. We talk here about how to deal with a problem which does not have a solution. This is a fundamentally profound question.
    - (C.J. Junta)

- We do not constrain you. Feel free to do the impossible.
  - (J.V. Ivanov)

Let’s assume that we can:

- Establish the existence of qualitatively different regimes of operation of large quantum coherent structures
- Identify universal dimensionless parameters controlling transitions between these regimes
- Develop reliable and efficient methods of calculation and measurement of these parameters
- Investigate the behaviour of small-scale model systems
- Extrapolate the results to large-scale systems

FOM candidates:

- Must be independent dimensionless combinations of world constants and characteristic parameters of a system of qubits, such as:

Dimensionality								
1	N	Z	$\langle \Delta j^2 \rangle$	…				
E^x	$\langle E \rangle$	$\langle \Delta E^2 \rangle$	$\langle g \rangle$	$\langle \Delta g^2 \rangle$	$\langle E_{ext} \rangle$	$\langle \Delta E_i \Delta E_j \rangle$	T	…
T^y	t_r	t_ϕ	t_{op}	$1/\dot{\lambda}$	$\langle \Delta t_r^2 \rangle$	$\langle \Delta t_\phi^2 \rangle$	…	
A^2T	$S_A(\omega)$	…						

Quantum speed limits

- Margolus-Levitin theorem
  - Transition between two mutually orthogonal states of a quantum system with average energy $\langle E \rangle$ (measured from the ground state) requires no less time than

$$\tau_{\perp} = h/4\langle E \rangle$$

  - N. Margolus, L.B. Levitin, Physica D **120**, 188 (1998)

Quantum speed limits

- Generalization 1:
  - In a driven closed system the transition from (pure) ρ_i to (mixed) ρ_f requires at least

$$\tau_{if} = \hbar/2\bar{E}\,\sin^2\Theta_{if}$$

- Bures angle $\Theta_{if} \equiv \arccos\sqrt{\langle\psi_1|\rho_f|\psi_1\rangle}$
- Average energy $\bar{E} = \frac{1}{\tau_{if}}\int_0^{\tau_{if}}\langle H(t)\rangle\,dt$
  - S. Deffner, E. Lutz, PRL **111**, 010402 (2013)

Quantum speed limits

- Generalization 2:
  - In an open system, where $\dot{\rho}_t = L_t[\rho_t]$, the transition requires at least

$$\tau_{if} = \max\{\frac{1}{\Lambda_{op}}, \frac{1}{\Lambda_{tr}}\} \sin^2 \Theta_{if}$$

    - $\Lambda_{op,tr} \equiv \frac{1}{\tau_{if}} \int_0^{\tau_{if}} \| L_t[\rho_t] \|_{op,tr} \, dt$
    - Here the *operator (trace)* norm is the largest singular value (their sum)
      - S. Deffner, E. Lutz, PRL **111**, 010402 (2013)

Quantum speed limits

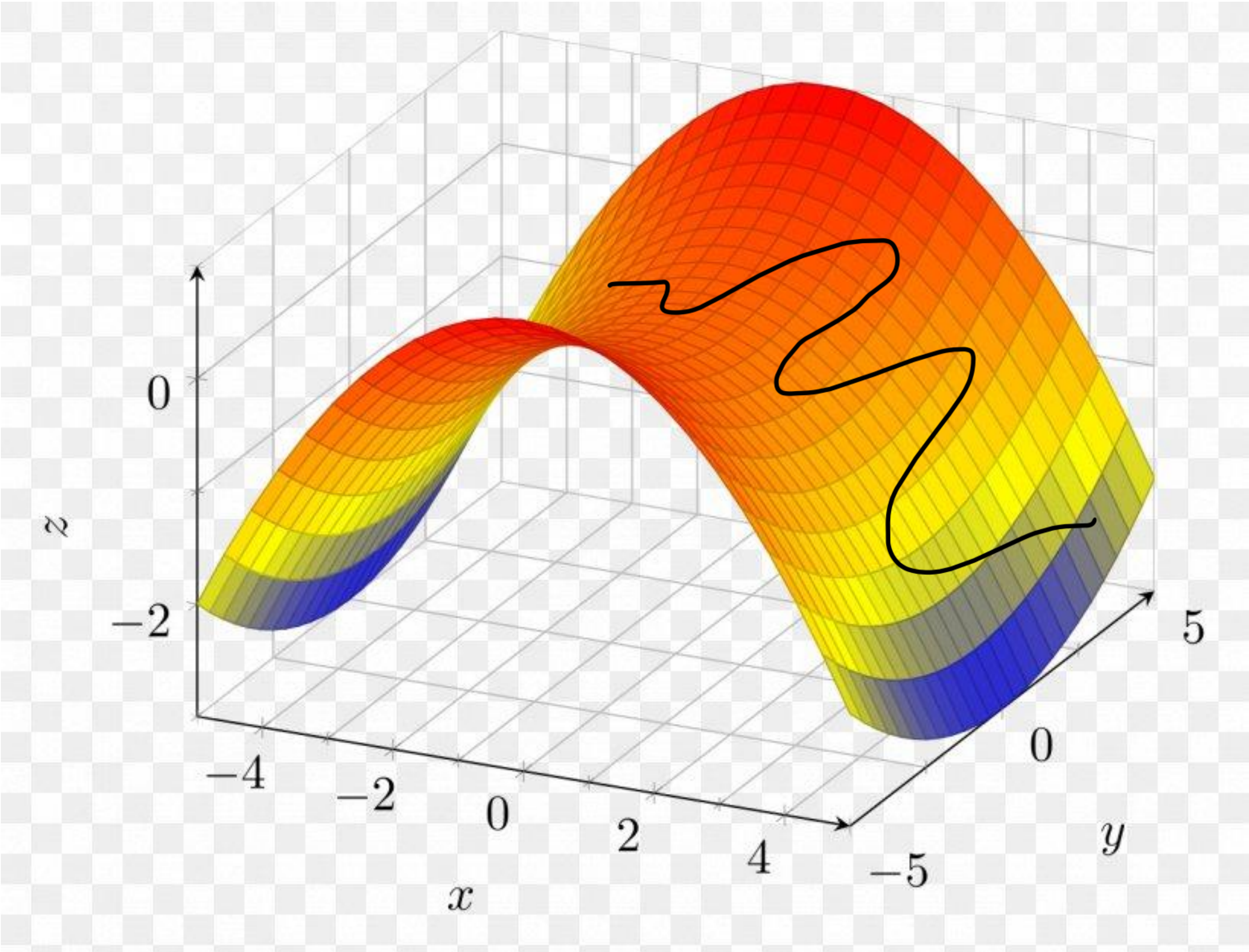
- Mandelstam-Tamm limit:
  - In an open system, where $\dot{\rho}_t = L_t[\rho_t]$, the transition requires at least

$$\tau_{if} = \max\{\frac{1}{\Lambda_{op}}, \frac{1}{\Lambda_{tr}}\} \sin^2 \Theta_{if}$$

    - $\Lambda_{op,tr} \equiv \frac{1}{\tau_{if}} \int_0^{\tau_{if}} \|L_t[\rho_t]\|_{HS}\, dt$
    - Here the Hilbert-Schmidt norm $\|L_t\| = \sqrt{tr[L_t L_t^{+}]}$
      - S. Deffner, E. Lutz, PRL **111**, 010402 (2013)

Quantum space limits

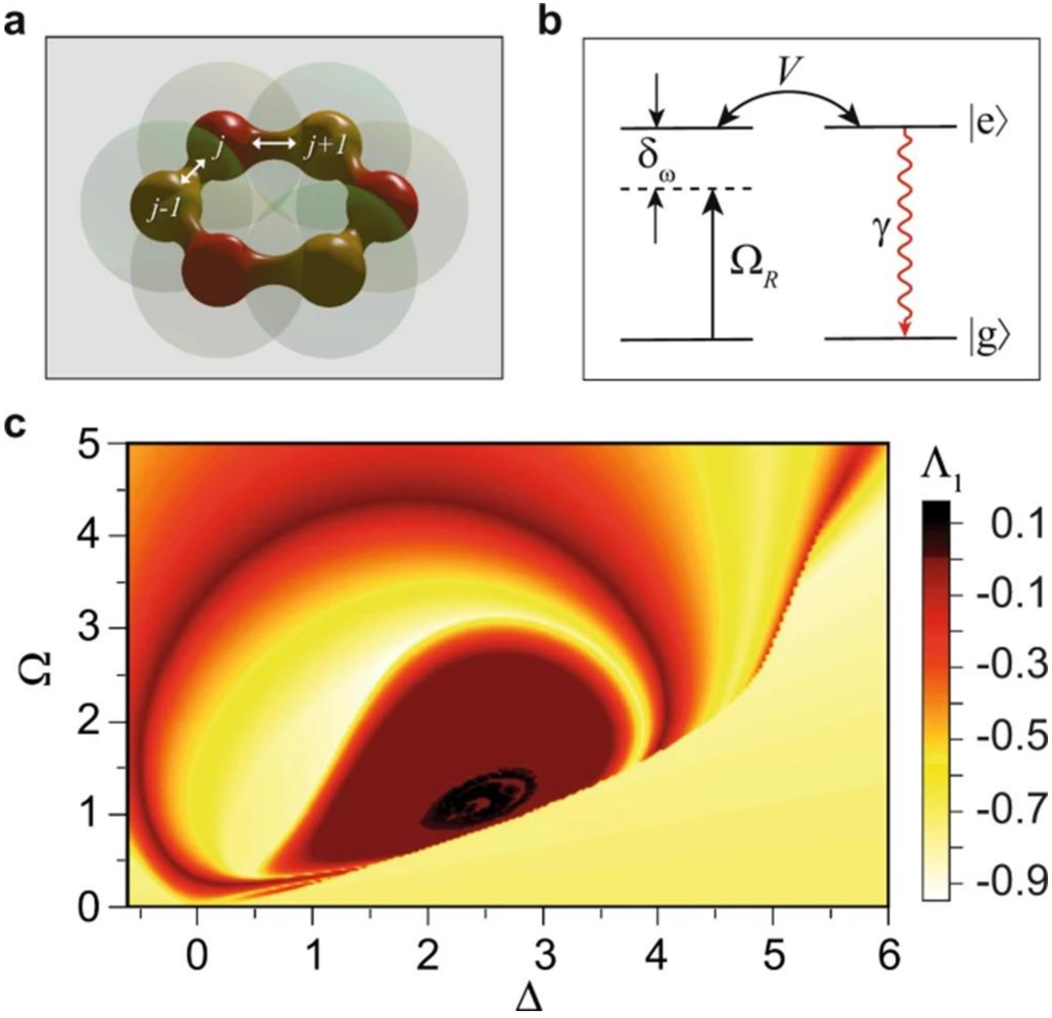
- "The current numerical and analytic evidence from the study of integrable systems suggests that there exists a minimal set of independent constraints the size of which is much less than the dimension of the Hilbert space but may still be much greater than one"
  - M. Rigol, V. Dunjko, M. Olshanii, Nature **452**, 854 (2008)


0
−2
z
−4
−2
0
2
4
x
−5
0
5
y

Quantum space limits

- The manifold of all quantum many-body states that can be generated by arbitrary time-dependent local Hamiltonians in a time that scales polynomially in the system size occupies an exponentially small volume in Hilbert space
  - $H(t) = \sum_{X \subset \{1,2,\ldots N\}} H_X(t); \quad \|H_X(t)\| \leq E$
  - X contains no more than k elements, independent of the system size

- D. Poulin, A. Qarry, R. Somma, F. Verstraete, PRL **106**, 170501 (2011)



A. Andreev, A. Balanov, T. Fromhold, M. Greenaway, A. Hramov, W. Li, V. Makarov and A. Zagoskin "**Chaos and hyperchaos in driven interacting quantum systems", npj Quantum Information 7, Article 1 (2021)**

$$\dot{\rho} = -i[H, \rho] + \mathcal{L}[\rho]$$

$$H = \sum_{j=1}^{N} \left[-\delta_\omega |e\rangle\langle e|_j + \frac{\Omega_R}{2} \left(|e\rangle\langle g|_j + |g\rangle\langle e|_j \right) \right] + \sum_{j<k} V_{i,j} |e\rangle\langle e|_j \otimes |e\rangle\langle e|_k ,$$

$$\mathcal{L}[\rho] = \gamma \sum_j \left[|g\rangle\langle e|_j \rho |e\rangle\langle g|_j - \frac{1}{2} \{ |e\rangle\langle e|_j , \rho \} \right]$$

$$\dot{w}_j = -2\Omega \Im q_j - (w_j + 1);$$

$$\dot{q}_j = i\left[\Delta - c\sum_{k\neq j}(w_k + 1)\right]q_j - \frac{1}{2}q_j + i\frac{\Omega}{2}w_j$$

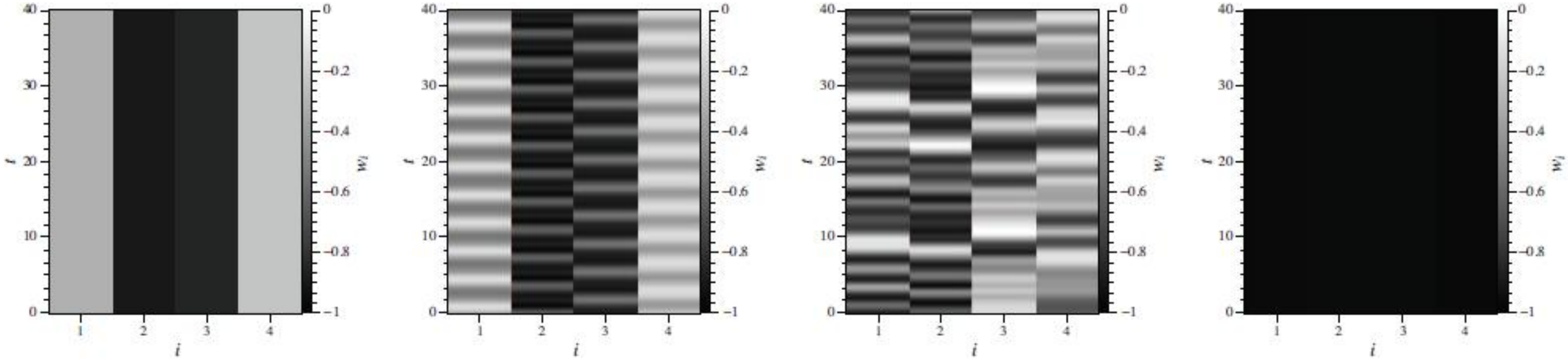


FIG. 1: Population inversion dependence on time (4 qubits) at different values of average qubit bias E; for a given qubit $E_j = E + \delta E_j$. Qubit parameters are given in the table 1. Left to right: $E = 1, 1.5, 3, 5.55$. We see the transition from periodic time dependence to stationary state, chaotic state and another stationary state.

j	δE_j	Δ_j	$J_{j-1,j}$	$J_{j,j+1}$
1	0.04	1.52000	0	4.9300000
2	-0.05	1.5200	4.97000	4.950000
3	0.03	1.57000	4.10000	5.0500
4	-0.02	1.43000	5.005000	0

TABLE I: Qubit parameters (Fig.1).

j	δE_j	Δ_j	$J_{j-1,j}$	$J_{j,j+1}$
1	0.04	1.52000	0	4.9300000
2	-0.05	1.5200	4.97000	4.950000
3	0.03	1.57000	4.10000	5.0500
4	-0.02	1.43000	5.005000	5.07000
5	0.07	1.50	5.1	0

TABLE II: Qubit parameters for 5 qubits (Fig.2).

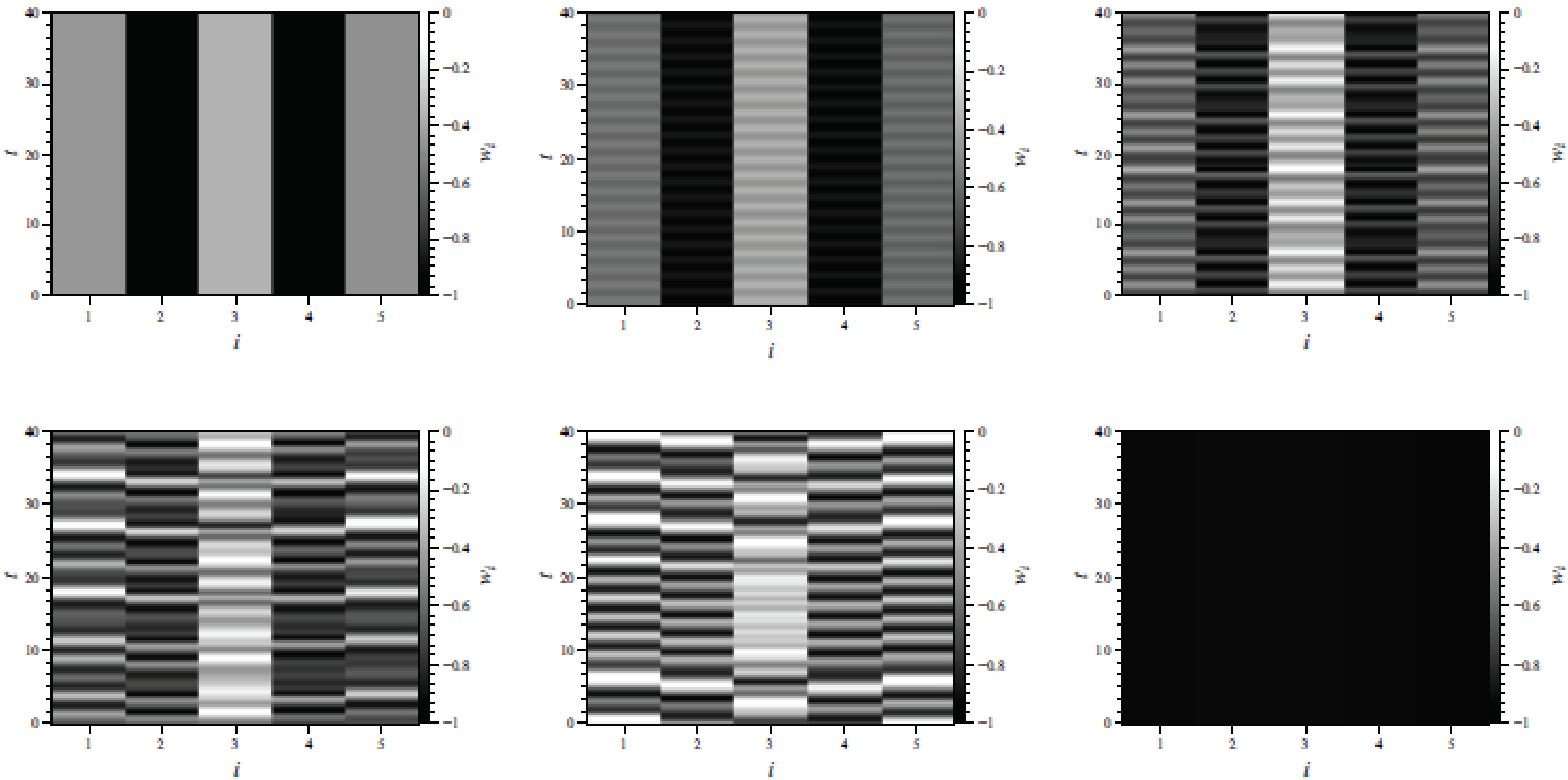
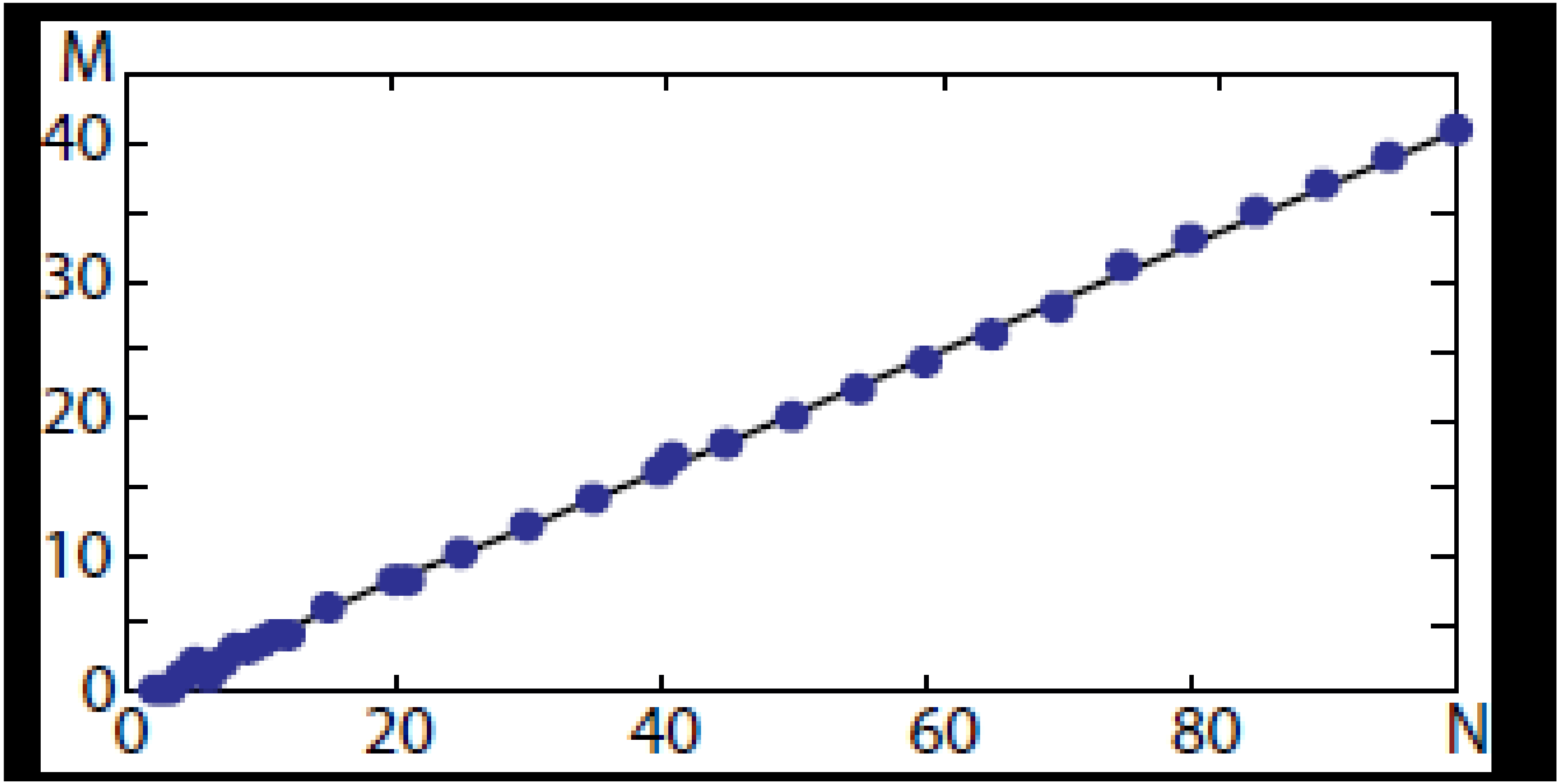
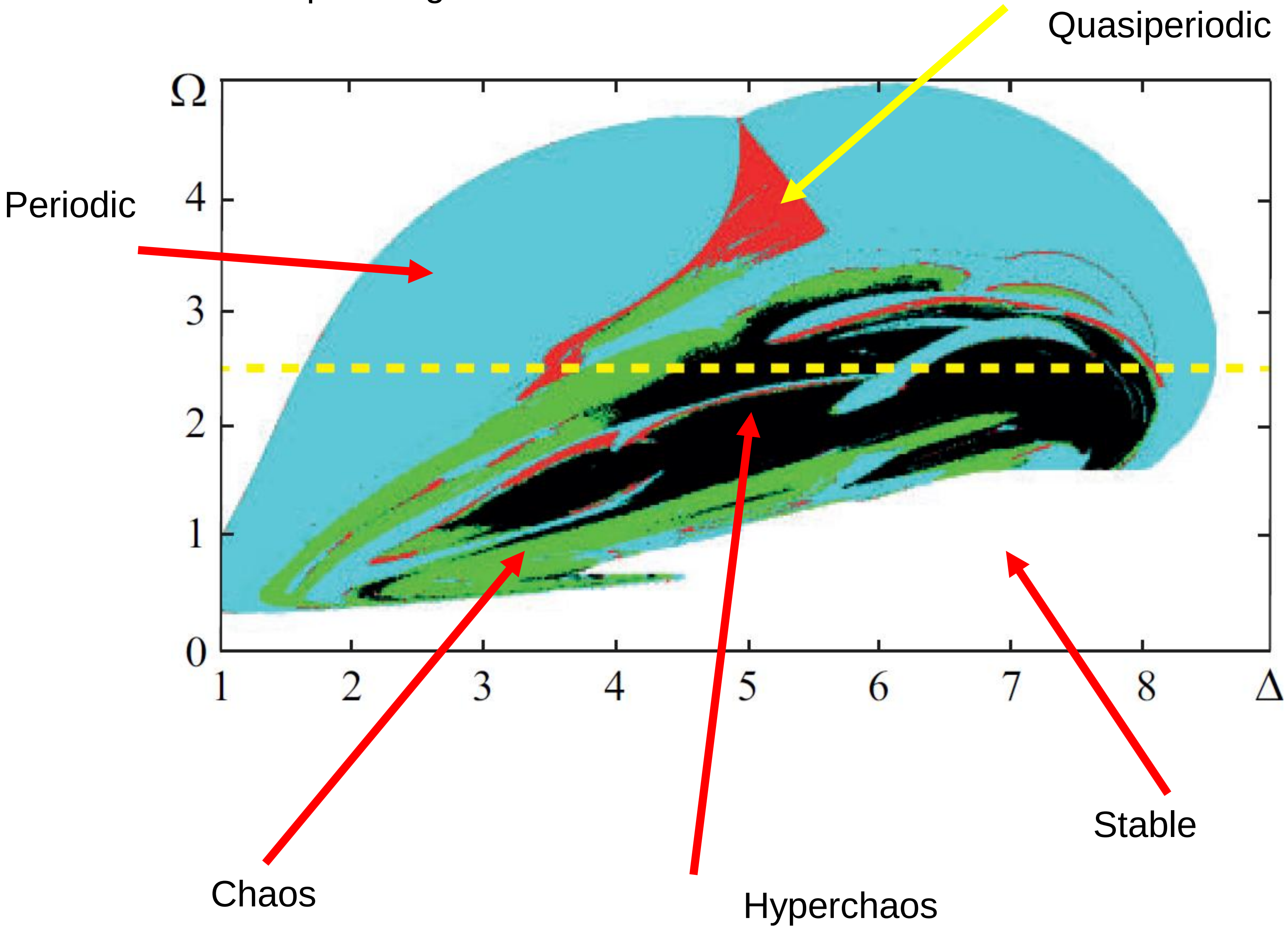


FIG. 2: Population inversion dependence on time (5 qubits) at different values of average qubit bias E; for a given qubit $E_j = E + \delta E_j$. Qubit parameters are given in the table 1. Left to right: $E = 1, 1.6, 2.1, 2.85, 3.48, 4.262$. Here the transition is from a stationary state through periodic time dependence to stationary state, then two hyperchaotic states and another stationary state.

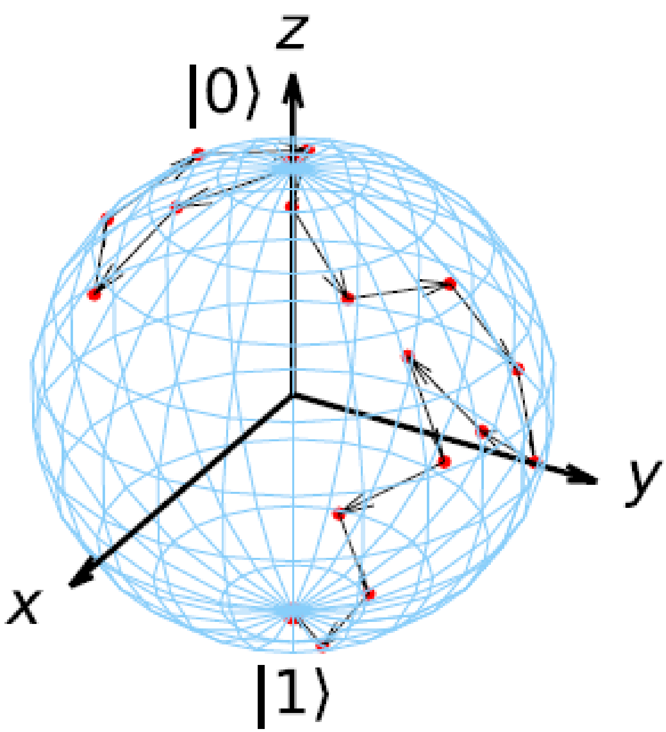
Number of positive Lyapunov exponents in
An N-qubit ring chain



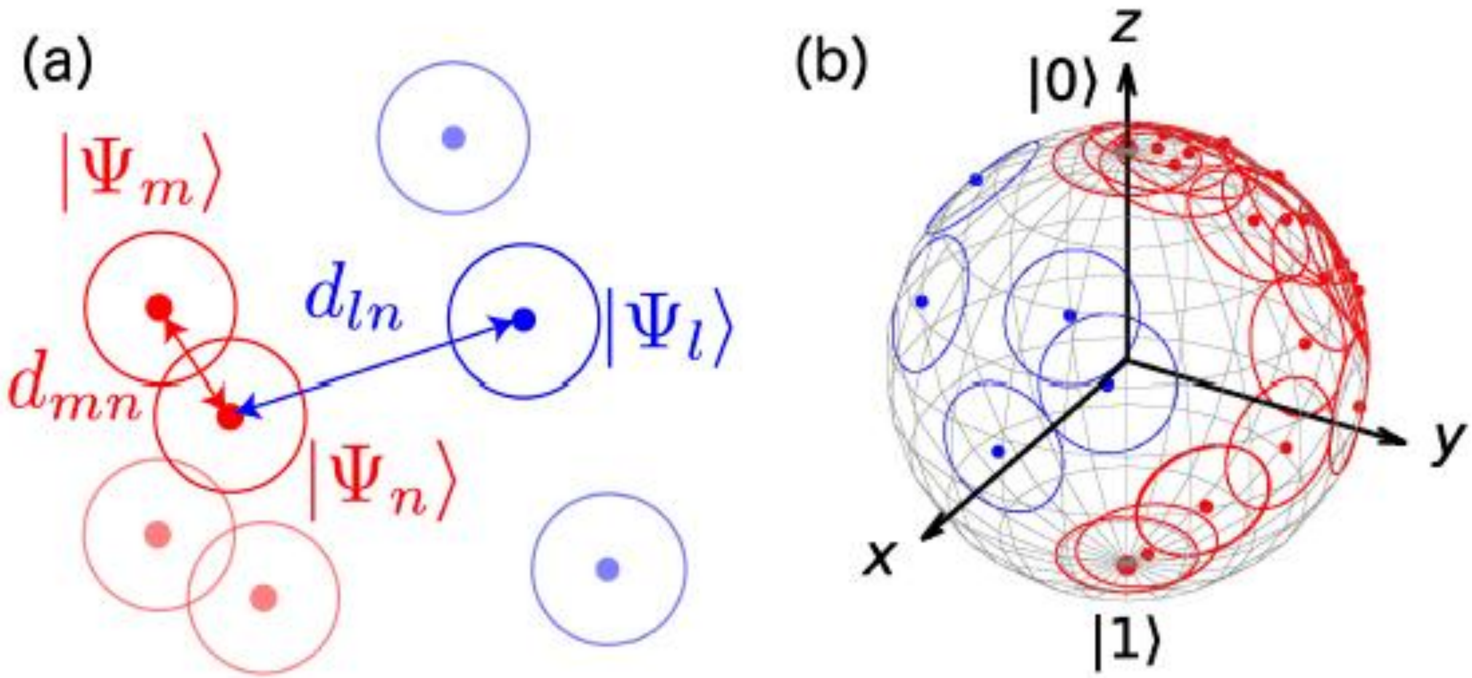
Five-qubit ring chain



Quantum evolution as a series of unitary evolutions interrupted by collapses



- **From where to where?**
  - ex) single qubit
    - from north pole $|0\rangle$ to south pole $|1\rangle$
  - N-qubit system
    - from one initial state $|\mathrm{i}\rangle$
      to one of the orthogonal state $|\mathrm{f}\rangle$
- **How to characterize?**
  - Fubini-Study distance: $d = \cos^{-1}(|\langle \mathrm{f}|\mathrm{i}\rangle|)$
    - $d \in [0, \pi/2]$
    - If $|\mathrm{i}\rangle$ and $|\mathrm{f}\rangle$ are orthogonal, then $d = \dfrac{\pi}{2}$.



$$\cos s\,(\phi, \psi) = \sqrt{\frac{\langle\phi|\psi\rangle\langle\psi|\phi\rangle}{\langle\phi|\phi\rangle\langle\psi|\psi\rangle}}.$$

$$\mathcal{S} = \frac{2}{\pi}\sqrt{s^2\,(\phi, \psi)}$$

$$s \approx \frac{\Delta t}{\hbar}\sigma_{\hat{H}}(\psi) \qquad \Delta t_{\max} = \frac{\pi}{2}\frac{\hbar}{\sigma_{\hat{H}}(\psi)}.$$

$$s(\phi,\psi) \approx \frac{\Delta t}{\hbar}\sigma_{\sum_j \hat{H}_j}(\psi), \qquad \sqrt{\overline{s(\phi,\psi)^2}} \approx \Delta S \frac{\sqrt{\overline{\sigma^2_{\sum_j \hat{H}_j}(\psi)}}}{\overline{\sigma_{\hat{H}}(\psi)}},$$

$$\sigma_{\sum_j \hat{H}_j}(\psi) = \left\langle\left(\sum_j \hat{H}_j\right)^2\right\rangle_\psi - \left\langle\sum_j \hat{H}_j\right\rangle_\psi^2$$

$$= \sum_{\chi\neq\psi}\sum_{jk}\langle\psi|\hat{H}_j|\chi\rangle\langle\chi|\hat{H}_k|\psi\rangle.$$

$$\Delta S = \overline{\sigma_{\hat{H}}} \Delta t/\hbar.$$

$$\sqrt{\overline{s(\phi,\psi)^2}} \approx \Delta S \frac{\sqrt{\overline{\sigma^2_{\sum_j \hat{H}_j}(\psi)}}}{\overline{\sigma_{\hat{H}}(\psi)}},$$

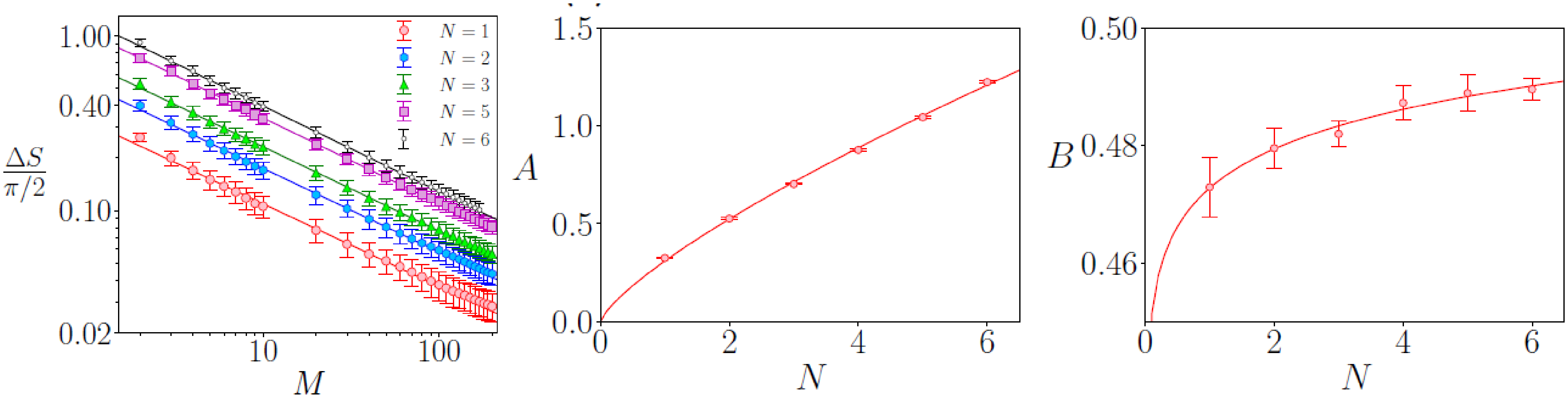
$$\overline{\sigma^2_{\sum \hat{H}_j}(\psi)} = \sum_{j=1}^{M} \sum_{\chi \neq \psi} \overline{|\langle \psi | \hat{H}_j | \chi \rangle|^2}$$

$$\approx M f(N) \overline{\sigma_{\hat{H}}(\psi)}^2.$$

$$\mathcal{S} \approx \frac{2}{\pi} \Delta S M^{1/2} f(N)^{1/2}.$$

$$\boldsymbol{S = 1 \rightarrow \Delta S = \frac{\pi}{2} M^{-\frac{1}{2}} f(N)^{-\frac{1}{2}}}$$

Random walk model: numerical results



- $\Delta S = \frac{\pi}{2} A(N) M^{-B(N)}$
  - $A(N) = 0.309\, N^{0.76}$
  - $B(N) = 0.473\, N^{0.020}$
    - $\rightarrow f(N) \approx 10.5\, N^{-\frac{3}{2}}$

Watabe, Serikow, Kawabata, and Zagoskin
Frontiers in Physics, 2022 (744).

Accessibility index

- Total evolution time t_f, decoherence time t_D, coupling J:
  - $M = \frac{t_f}{t_D}$; $\Delta S \leq t_D J$
- The necessary "quantumness condition" is $A>1$, where the *accessibility index*
- $A = 4J\ \frac{\sqrt{t_D t_f}}{N^{\frac{3}{4}}}$

Quantumness of a D-Wave processor

- $N \sim 100$
- $t_D \sim 10$ ns
- $t_f \sim 5\mu s$
- $J \sim 5$ GHz

- $A \sim 100 \gg 1$
- The maximal size of a quantum adiabatic processor (with these parameters):
- $N_m = (4J)^{\frac{4}{3}}\left(t_f t_D\right)^{\frac{2}{3}} \sim 10^5$

Percolation approach

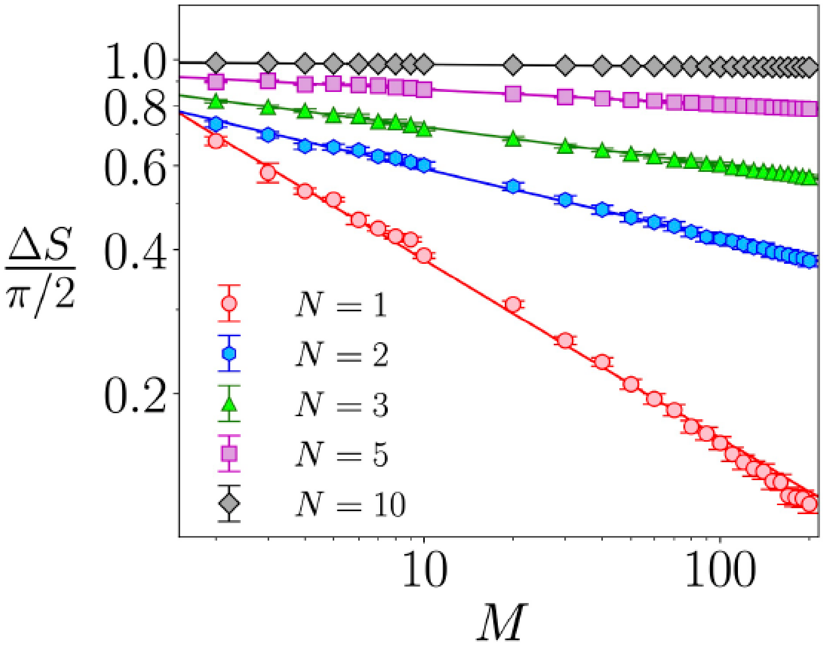


FIG. 2. The Fubini-Study threshold distance ΔS for connecting two states as a function of the number of isotropic random states M in the Hilbert space for N-qubits, where the cluster can expand the maximum Fubini-Study distance $\pi/2$. The data points are the averaged value for 100 samples, and solid lines are fitted lines with $\Delta S = (\pi/2)AM^{-B}$ for $M = 2$–200.

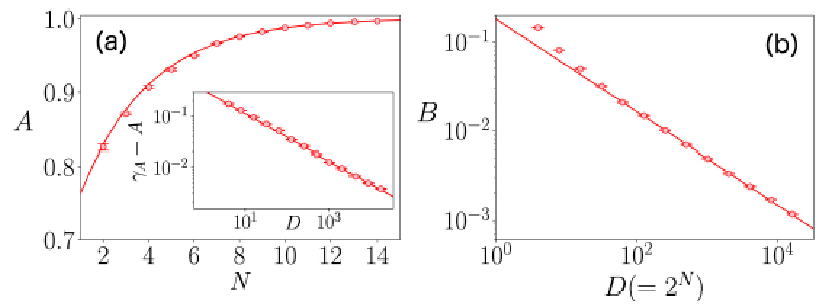


FIG. 3. (a) The fitting factor A and (b) the fitting factor B as a function of the number of qubits N and the dimension of the Hilbert space: $D = 2^N$. The solid lines represent fitting functions, where we used $A = \gamma_A - \alpha_A D^{-\beta_A}$ and $B = \alpha_B D^{-\beta_B}$ for $N = 7$–14. The inset in (a) is a log-log plot of $\gamma_A - A$ as a function of D.

There are indications that we actually can:

- Establish the existence of qualitatively different regimes of operation of large quantum coherent structures
- Identify universal dimensionless parameters controlling transitions between these regimes
- Develop reliable and efficient methods of calculation and measurement of these parameters
- Investigate the behaviour of small-scale model systems
- Extrapolate the results to large-scale systems

In science fiction radio was the main thing. It was expected to bring universal happiness to the mankind.
Well, now we have radio, but still no happiness in sight.

Ilya Ilf (ca. 1930)

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Funded by
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QRC-4-ESP